Problems from the upcoming 7th Edition of *Applied Calculus* by Hughes Hallett, et al.

Sample problems – Section on Linear Functions

1. Facebook's annual ad revenue increased every year from 2011 through 2018, and each year the increase in revenue was larger than the year before.[1] Was the growth in revenue linear?
2. Let t be in quarter-years after the last quarter of 2018. Zoom,[2] the video conferencing company, had quarterly revenue R through $t = 5$ of about

$$R = 44.6 + 15.5t \text{ million dollars}$$

Then, from $t = 6$ onward in 2020, quarterly revenue was about

$$R = 11.6 + 22.1t \text{ million dollars}$$

(a) Interpret the slope of each function.
(b) What might have caused the change in slope at the time it happened?

3. Figure 1 shows the 0–60 mph acceleration time, A (in seconds), for the best-accelerating cars[3] of the year, t years after 1955, where $A = -0.0566t + 6$.

(a) What are the units and meaning of the

(i) Slope? (ii) Vertical intercept?

(b) In what year is the 0–60 mph acceleration time predicted to drop to 1 second?

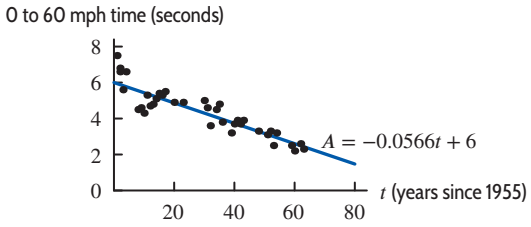


Figure 1

Sample problems – Section on Average Rate of Change and Relative Change

4. Figure 2 shows graphs of US advertising revenue (in billions of dollars) for print newspapers and for the internet[4] as a function of years since 1997.

(a) Which graph, A or B, is the graph of newspaper ad revenue? Explain how you know.
(b) Find the average rate of change of newspaper ad revenue and of internet ad revenue between 1997 and 2016. Include units and explain the signs.

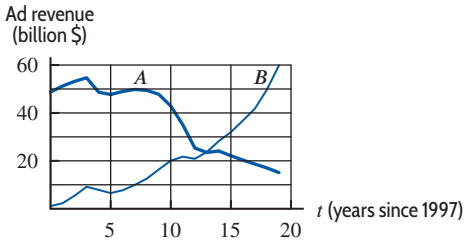


Figure 2

5. Let $P(t)$ be the total number of Covid-19 cases in Minnesota confirmed up to and including day t, where $t = 0$ is March 5, 2020.[5] Let $N(t)$ be the number of new cases confirmed on day t. What does each statement mean in the context of the outbreak?

(a) $P(37) - P(36) = 80$
(b) $(P(37) - P(36))/P(36) = 0.074 = 7.4\%$
(c) $(P(37) - P(36))/P(37) = 0.069 = 6.9\%$
(d) $P(35) + N(36) + N(37) = 1164$

[1] https://i1.wp.com/www.mekkographics.com/wp-content/uploads/2019/03/Slide39.png
[2] news.alphastreet.com/infographic-zoom-video-communications-zm-q2-2021-earnings-results, accessed March 23, 2021.
[3] Cars tested by *Car and Driver* Magazine, https://www.caranddriver.com/features/g15383525/car-and-driver-tested-the-quickest-cars-from-the-1950s-to-today/?slide=4, accessed January 9, 2021.
[4] Based on 2017 research by David Flath using www.journalism.org/fact-sheet/newspapers, www.iab.net, and fred.stlouisfed.org/series/GDPDEF.
[5] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed January 17, 2021.

Sample problems – Section on Exponential Functions

6. In 2020, Arizona's second wave of Covid-19 infections grew at 3% a day; that is, the number of confirmed cases grew by a factor of $(1.03)^t$ over t days. Covid-19 confirmed cases in West Virginia in 2020 grew at 1.75% a day; that is, by a factor of $(1.0175)^t$ over t days.[6]

(a) By what factor did Arizona's cases grow over four months? How about West Virginia's cases?

(b) Notice that Arizona's daily percent growth rate is nearly double the daily percent growth rate in West Virginia. Is the 4-month growth factor for Arizona approximately double the 4-month growth factor for West Virginia?

7. On June 16, 2020, the New York Times wrote,[7] "The World Health Organization said last week that confirmed cases in Africa had doubled in 18 days to reach 200,000; the first 100,000 took 98 days."

(a) Make an exponential model for the total number of Covid-19 cases in Africa during the initial 98 days. Use the fact that on the first day there was one confirmed case in Africa.[8]

(b) How does your model's prediction for day 116 compare to the number 200,000 in the article?

Sample problems – Section on The Natural Logarithm

8. A quantity is growing exponentially: $P = P_0e^{kt}$, with P_0 and k constant.

(a) Show that a plot of $y = \ln P$ against t, with t on the horizontal axis, is a line.

(b) What is the slope of the line?

(c) What is its vertical intercept?

9. Use the results of Problem 8. Let P be the total number of Covid-19 cases in South Africa[9] confirmed up to day t, where $t = 0$ is March 18, 2020. Figure 3 shows $\ln P$ plotted against t.

(a) Estimate the continuous growth rates before and after the bend in the graph at about $t = 10$ days.

(b) What might have happened to lead to the bend in the graph?

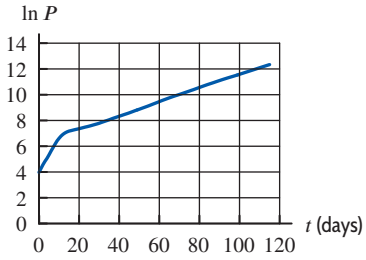


Figure 3

Sample problems – Section on Exponential Growth and Decay

10. The Outer Banks are a string of linked islands off the coast of North Carolina. Rising sea levels are eroding the beach and threatening the only road. Avon, a small community, is deciding how to pay for "beach nourishing," which builds up the beach with sand, but has to be repeated every 5 years. If the nourishings cost $12 million each and the first is in one year, what is the present value of a sequence of four beach nourishings?[10] Assume the interest rate is 1.5%, compounded annually.

11. In the Netherlands, a total of 337,245 Covid-19 cases had been confirmed by November 1, 2020. With t in days since November 1, the total number of cases, $P(t)$, was growing at 3.4% per day.[11]

(a) Find the total number of cases expected in the Netherlands by December 31, 2020 ($t = 60$) if the growth rate remained constant.

(b) Find a new growth factor a making the number of total cases on December 31, 2020 half of the number predicted in part (a).

[6] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed January 10, 2021.
[7] https://www.nytimes.com/2020/06/16/world/africa/coronavirus-africa.html accessed January 10, 2021.
[8] https://github.com/owid/covid-19-data/tree/master/public/data, accessed March 26, 2021.
[9] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed July 5, 2020.
[10] https://news.yahoo.com/tiny-town-big-decision-willing-121315761.html, accessed March 15, 2021.
[11] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed February 26, 2021.

Sample problems – Section on The Derivative Function

12. Table 1 shows the area, A, covered by Arctic sea ice in millions of square kilometers between 2017 and 2020.[12] Let $A = f(t)$ for t in years since 2000.

(a) Does $f'(t)$ appear to be positive or negative? Interpret in terms of Arctic sea ice.
(b) What is $f(17)$? Estimate $f'(17)$. Give units.
(c) What is $f(20)$? Estimate $f'(20)$. Give units.

Table 1

Year	2017	2018	2019	2020
Sea Ice (mn km^2)	4.82	4.79	4.36	3.92

Sample problems – Section on Interpretations of the Derivative

13. Johns Hopkins University reports two numbers daily for Covid-19 cases in each US state:[13]

- $P(t)$: the total number of cases confirmed up to time t, in days
- $N(t)$: number of new cases confirmed on day t.

(a) What are the units of $P'(t)$?
(b) Explain the meaning of $P'(t)$ in the context of the pandemic.
(c) Explain why $N(t)$ and $N(t+1)$ are good approximations for $P'(t)$.

14. Figure 4 shows the total number $P(t)$ of Covid-19 cases in New York confirmed on or before day t, where $t = 0$ is March 15, 2020. Figure 5 shows $N(t)$, the new cases on day t for New York, Florida, Maine and Wyoming.[14] Use the fact that $dP/dt \approx N$ to identify the $N(t)$ graph for New York.

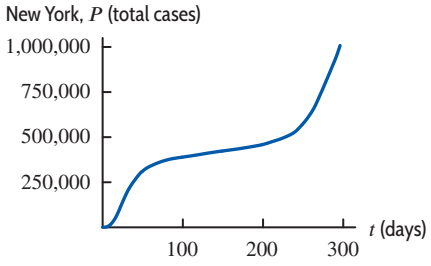


Figure 4

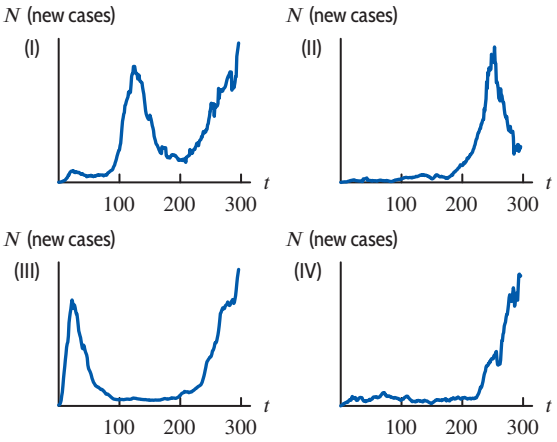


Figure 5

15. The range $R = f(T)$ of an electric vehicle, the average distance (in miles) it is expected to travel on a fully charged battery, depends on the outside temperature T (in degrees Fahrenheit). For a 2014 Nissan Leaf[15] $f(50) = 72$ and $f'(50) = 0.6$.

(a) What are the units and meaning of
(i) $f(50) = 72$? (ii) $f'(50) = 0.6$?
(b) What is an approximate range for the Leaf at 55°F?

Sample problems – Section on Marginal Cost and Revenue

16. Figure 6 shows marginal revenue, MR, as a function of quantity, q, of an item produced. Match each graph (I)–(IV) with one of the statements (a)–(d).

(a) Constant price
(b) As quantity increases, price decreases and then increases
(c) As quantity increases, price decreases
(d) Price increasing with quantity

[12] https://climate.nasa.gov/vital-signs/arctic-sea-ice/, accessed 3 April, 2021. To facilitate yearly comparisons, the values shown were all in September, usually the lowest point of the year.
[13] https://coronavirus.jhu.edu/us-map
[14] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed October 22, 2020.
[15] https://blog.ucsusa.org/dave-reichmuth/electric-cars-cold-weather-temperatures , accessed on January 11, 2021

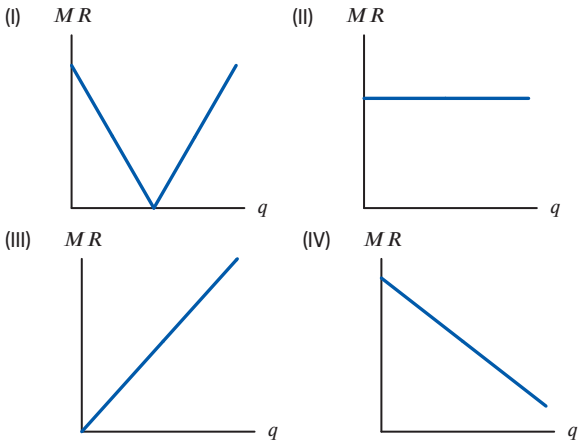


Figure 6

Sample problems – Section on Using the Derivative

17. Figure 7 shows the total number $P(t)$ of Covid-19 cases in Arizona confirmed on or before day t, where $t = 0$ is March 1, 2020;[16] $N(t)$ is the number of new cases on day t, approximated by $P'(t)$.

(a) Which of the following are the approximate t-values of local maxima of $N(t)$? (Select all that apply.) $t = 0$, 110, 130, 150, 190, 260, 320, 360.

(b) Which, from the previous list, are the approximate t-values of local minima of $N(t)$?

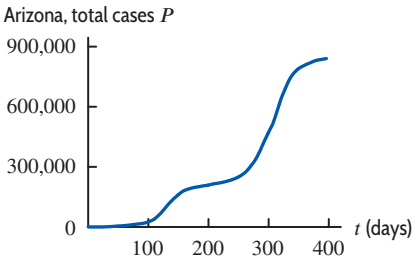


Figure 7

Sample problems – Section on Inflection Points

18. Figure 8 shows the total number $P(t)$ of Covid-19 cases (in millions) in California[17] on or before day t, where $t = 0$ is April 1, 2020. Estimate the t-values of two inflection points and explain their significance in terms of the number of daily new cases, approximated by $P'(t)$.

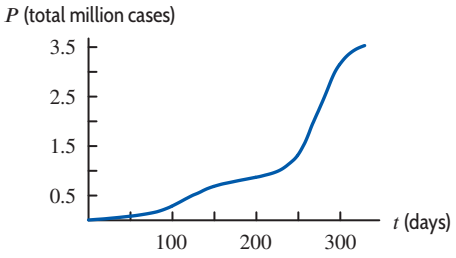


Figure 8

[16] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed March 24, 2021.
[17] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed March 1, 2021.

19. The range R of an electric vehicle, the average distance (in miles) it is expected to travel on a fully charged battery, depends on the outside temperature T (in degrees Fahrenheit). Figure 9 shows $R = f(T)$ for the 2014 Nissan Leaf.[18]

(a) At what temperatures does the graph have inflection points?

(b) At which temperature does an increase in temperature add the most to the range of the Leaf? How is this temperature related to the inflection points?

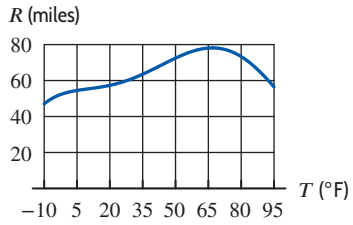


Figure 9

Sample problems – Section on Analyzing Antiderivatives Graphically and Numerically

■ In Problems **20–21**, graphs (I)–(IV) show Covid-19 data from two different countries.[19] For each country, one graph is of $P(t)$ and one graph is of $N(t)$, where

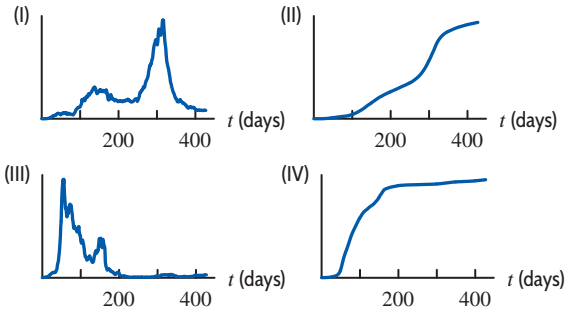
- $P(t)$: total number of cases on or before day t
- $N(t)$: number of new cases confirmed on day t.

Use the fact that $P(t)$ is an antiderivative of $N(t)$ since $P'(t) \approx N(t)$.

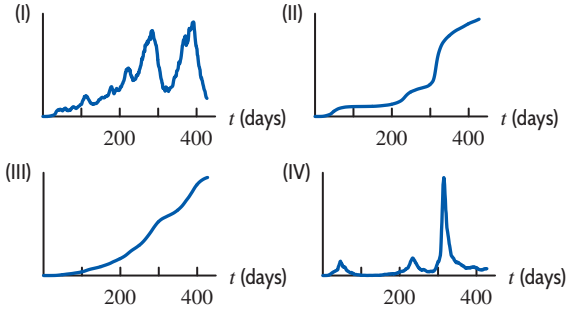
(a) Which of graphs (I)–(IV) are graphs of $P(t)$?

(b) Match each $P(t)$ with its corresponding $N(t)$.

20.



21.



[18] https://blog.ucsusa.org/dave-reichmuth/electric-cars-cold-weather-temperatures, accessed January 11, 2021.

[19] Smoothed from JHU CSSE COVID-19 data at github.com/CSSEGISandData/COVID-19, accessed May 3, 2021.