# Introduction

**$P$ and $N$ functions.** The cumulative total number of Covid-19 cases confirmed up to and including day $t$ is denoted by $P(t)$. We allow $t = 0$ to correspond to any given date.

The number of new cases confirmed on $N$ on day $t$ is given by

$$N(t) = P(t) - P(t-1).$$

We sometimes also call $N(t)$ the...

- daily increase in cases on day $t$.
- daily cases on day $t$.
- new cases.

Note that with these definitions

$$\begin{aligned} P(t) &= P(t-1) + N(t) \\ &= (\text{Total cases confirmed up to and including day } t) + (\text{New cases confirmed on day } t) \end{aligned}$$

**Data sources.** All data is sourced from the *COVID-19 Data Repository by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University* accessible at the following link

- https://github.com/CSSEGISandData/COVID-19.

We reference this source as JHU in short.

**Getting the data for your own explorations, or for students to explore.** Raw data for countries and US states, together with their 7-day smoothing (see below) can be downloaded using a couple of websites that Isaac Flath built for the Consortium.

- https://isaac-flath.shinyapps.io/coronavirus2/
- https://isaac-flath.shinyapps.io/Covid-19/

These two sites also generate plots for any given time range for all the quantities we discuss in the problems we have written ($P$, $N$, relative change of $P$, log plots of $P$, etc).

**Data smoothing.** Covid-19 data (specially $N$-data) is very noisy, so in all the problems we have written we use the 7-day moving average of the data which we compute as follows ($\widetilde{P}(t)$ is the raw time series obtained JHU ):

$$\begin{aligned} P(t) &= (\widetilde{P}(t) + \widetilde{P}(t-1) + \ldots + \widetilde{P}(t-6))/7 \\ N(t) &= P(t+1) - P(t) \end{aligned}$$

so, the new cases time series $N(t)$ gets smoothed through the smoothing of $P(t)$ and does not get smoothed separately. This makes the relation $N(t) = P(t+1) - P(t)$ continue to hold after the smoothing. We also make sure that we pick up JHU data well before $t = -7$, so the data is smoothed using the above formula even for $t = 0$ using the previous 7 data points in the JHU time series.

Figures 1 and 2 show the effect of this smoothing for the cumulative cases $P$ and new cases $N$ for Romania, with $t$ in days since March 1, 2020 (thick blue is the smoothed data, thin black is the raw data). A side product of this smoothing is that data gets shifted forward (to the left) by 3.5 days.

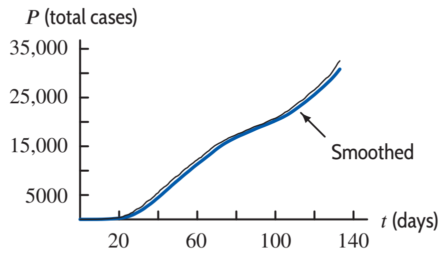


Figure 1

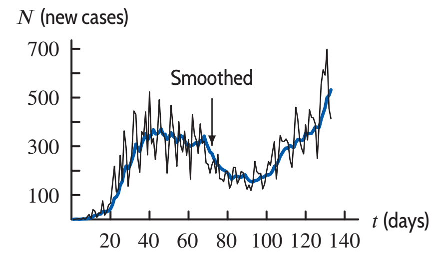


Figure 2

We will not use raw data $\widetilde{P}(t)$ (or $\widetilde{N}(t) = \widetilde{P}(t+1) - \widetilde{P}(t)$) in what follows unless explicitly stated otherwise.

**Expected shape of $P$ and $N$.** The Covid-19 outbreaks have occurred in waves in many countries. Each wave has an $N$-plot which is bell-shaped and that will correspond to a sigmoid shaped $P$-curve.

Subsequent waves stack on top of one another and result in $N$-plots that look like a sum of bell-shaped curves (which may or may not drop all the way down to zero between the bells), and this results in a $P$-plot that will look like a sequence of stacked sigmoid s-curves. Figures 3 and 4 are the $P$ and $N$ plots for Australia between March 1 and August 16, 2020.

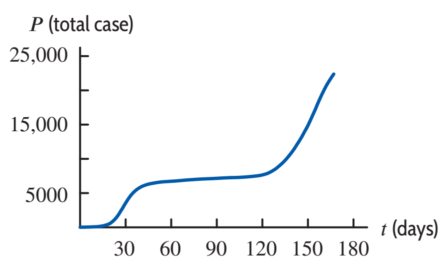


Figure 3

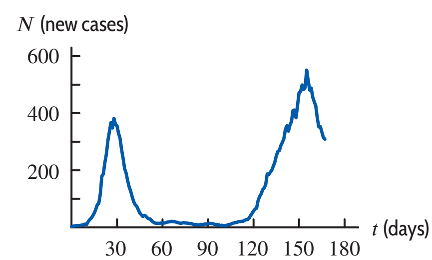


Figure 4

Of course, the outbreaks in many countries do not look so distinctly as a sum of waves. Besides possible data collecting issues or changes in data collecting strategies, drastic measures implemented to control the outbreak have noticeable effects on the way the outbreak developed. Figures 1 and 2 are good examples of expected shapes $P$ and $N$ plots when the waves are not separate.

**Many more problems and problem ideas in the Instructor notes package.** Visit `https://mcwg.github.io/covid` for many more problems and problem ideas.

## Sample problems - Precalculus - see the Instructor notes package for more

**1.** Figure 5 and 6 show data from the Covid-19 outbreak in Switzerland[1] during the Spring of 2020. Which graph is the graph of the function $P(t)$, total number of cases confirmed up to and including day $t$, and which is the graph of the function $N(t)$, number of new cases confirmed on day $t$?

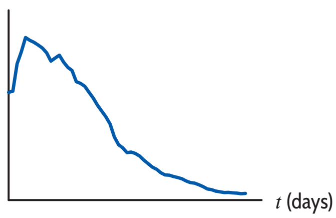


**Figure 5**

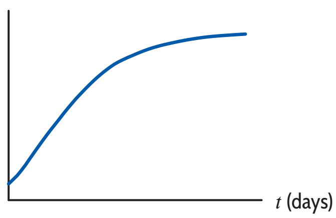


**Figure 6**

**2.** Let $P(t)$ be the total number of Covid-19 cases in Minnesota confirmed up to and including day $t$, where $t = 0$ is March 5, 2020.[2] Let $N(t)$ be the number of new cases confirmed on day $t$. Explain the meaning of the statements in the context of the outbreak.

**(a)** $P(37) - P(36) = 80$
**(b)** $(P(37) - P(36))/P(36) = 0.074 = 7.4\%$
**(c)** $(P(37) - P(36))/P(37) = 0.034 = 3.4\%$
**(d)** $P(35) + N(36) + N(37) = 1164$

**3.** Between 3/26/20 and 4/15/20 Turkey experienced rapid growth in the number of reported Covid-19 cases.[3] Figure 7 shows the graph for the total cumulative number of Covid-19 cases, $P$, and Figure 8 shows its daily percent increase. Note that $P$ is increasing and concave up. Was $P$ growing exponentially between 3/26/20 and 4/15/20? How does the plot of the daily percent increase help you decide?

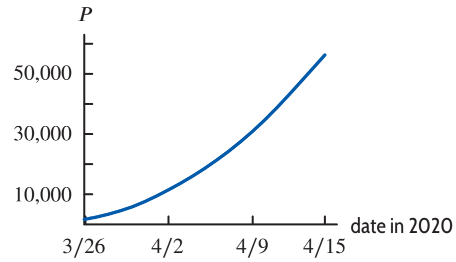


**Figure 7**

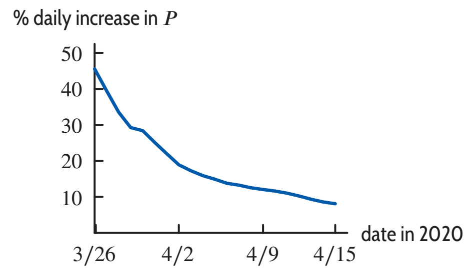


**Figure 8**

**4.** On June 16, 2020, the New York Times in its *Live Coronavirus Updates* section[4] stated that "The World Health Organization said last week that confirmed cases in Africa had doubled in 18 days to reach 200,000; the first 100,000 took 98 days." Assume that initially there was one confirmed case in Africa. Use an exponential model to represent the growth of total Covid-19 cases in Africa during the initial 116 days. How does your model's prediction compare to the number 200,000 in the article? Explain.

**5.** **(a)** Let $P(t)$ be the total number of cases confirmed up to and including day $t$ of an infectious disease outbreak. If every confirmed case goes into quarantine immediately for exactly 10 days, explain why the number of people in quarantine on day $t$ is given by

$$\text{people in quarantine} = P(t) - P(t - 10).$$

**(b)** How does the formula in part (a) change if only a fraction $r$ of total cases goes into quarantine and the length of the quarantine is $s$ days?

[1] Data from Johns Hopkins University, downloaded from https://github.com/CSSEGISandData/COVID-19 accessed on May 23, 2020.
[2] Data from Johns Hopkins University, downloaded from https://github.com/CSSEGISandData/COVID-19 accessed on January 17, 2021.
[3] Data from Johns Hopkins University, downloaded from https://github.com/CSSEGISandData/COVID-19 accessed on June 25, 2020.
[4] https://www.nytimes.com/2020/06/16/world/africa/coronavirus-africa.html accessed on January 10, 2021.

**6.** "Flattening the curve" in an epidemic means reducing the slope of the graph of total confirmed cases, $P$. Figure 9 shows a graph of $P(t)$ and $P(t-5)$, where $P(t)$ is the confirmed Covid-19 cases in a community on day $t$. A flattening of $P$ took place on day $t = T$.

If all confirmed cases are immediately checked into a hospital and spend 5 nights there, then the number of patients in the hospital on day $t$ is given by:

$$\text{People in hospital} = P(t) - P(t-5).$$

**(a)** Explain why the graphs of $P(t)$ and $P(t-5)$ have the same shape.
**(b)** What do the lengths of line segments $A$ and $B$ represent in terms of the epidemic and the hospital?
**(c)** The number of cases during the entire course of an epidemic might not be reduced by flattening the curve. Explain, by comparing the lengths of $A$ and $B$, why it is helpful to flatten the curve anyway, for example by widespread social distancing.

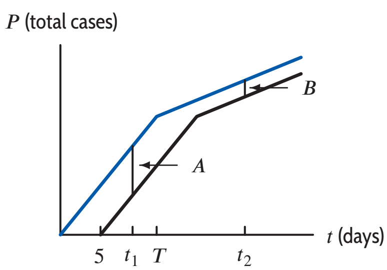


**Figure 9**

**7.** During the spring of 2021 a new Covid-19 variant from the UK started to spread in the US. Cases related to this variant were discovered to be doubling every 10 days.[5] How many days does it take for the number of cases related to this variant to multiply by 10? Choose among the numbers 28, 30, 33, 39, 45, 50.

**8.** The total number of confirmed Covid-19 cases in the US[6] between March 7, 2020 and March 17, 2020 is shown in Table 1.

**(a)** An exponential function can be used to model the total number of cases $t$ days after March 7, 2020. Which of the following daily growth rates would fit the model best? 28%, 33%, 37%. Explain.
**(b)** Using your answer to part (a) write an exponential function to model the total number of cases as a function of days since March 7, 2020.
**(c)** The actual reported number of cases on March 27 was 58,330. How does this number compare to your model's prediction on that day?
**(d)** The actual reported number of cases on April 30 was 988,487. How does this number compare to your model's prediction on that day?

**Table 1**

| Date in 2020 | Total Cases |
|---|---|
| March 7 | 190 |
| March 8 | 253 |
| March 9 | 323 |
| March 10 | 444 |
| March 11 | 606 |
| March 12 | 813 |
| March 13 | 1086 |
| March 14 | 1419 |
| March 15 | 1850 |
| March 16 | 2432 |
| March 17 | 3214 |

**9.** An epidemic is in an exponential growth phase when the number of cases of a disease increases by the same percent every day. When growing exponentially, there is a constant doubling time for the total number of cases. The doubling time is sometimes given in popular news reports because it is easy to understand what it means. An ancient rule of thumb, called the rule of 72, says that the doubling time is about $72/x$ days when growth is $x$% per day.

**(a)** An exact formula for the doubling time if growth is $x$% per day is given by

$$\text{Doubling time} = \frac{\ln 2}{\ln(1 + \frac{x}{100})} \text{ days.}$$

Use it to complete the table.

| Daily percent increase, $x$ | 1 | 5 | 7 | 10 | 15 | 20 |
|---|---|---|---|---|---|---|
| Doubling time (days) | 69.7 | | | | 5.0 | |
| $72/x$ (days) | 70 | | | | 4.8 | |

**(b)** The percent error between the exact doubling time and its rule of 72 approximation is given by

$$\text{Percent error} = \frac{(\text{Rule of 72 approximation}) - (\text{Exact value})}{\text{Exact value}} \times 100.$$

Use the formula to find the percent error in the rule of 72 for each growth rate in the table.
**(c)** Do you think the rule of 72 is accurate enough to be useful?

[5] https://www.medrxiv.org/content/10.1101/2021.02.06.21251159v1.full.pdf+html and https://www.webmd.com/lung/news/20210208/uk-covid-variant-doubles-every-10-days-in-the-us, accessed on February 11, 2021

[6] Data from Johns Hopkins University, and smoothed using a 7-day moving average. Downloaded from https://github.com/CSSEGISandData/COVID-19 accessed on April 26, 2020.

**10.** The total number of confirmed Covid-19 cases in Chile[7] during most of May 2020 can be modeled by $P(t) = P_0(1.06)^t$, where $t = 0$ is May 1, 2020, and $P(t)$ is the total number of cases up to and including day $t$. Show that $N(t)$, the number daily cases, given by $N(t) = P(t) - P(t-1)$, was also growing exponentially. At what rate was it growing?

**11.** The total number of confirmed coronavirus cases in Egypt and the US[8] grew at considerably different rates during the initial outbreak. For example, the number of confirmed cases in the US between between 3/6/20 and 3/16/20 grew approximately exponentially with a 32% per day growth rate. The initial outbreak in Egypt started later and the number of confirmed cases grew by approximately 5% per day between the time period 4/18/20 and 5/9/20.

**(a)** By what factor did the number of cases grow in the US in that 10-day period? How about Egypt in that 21-day period?

**(b)** Compute the doubling time for the number of confirmed cases in the US and Egypt during the periods in which they had 32% and 5% growth rate. Which one is smaller? Explain, why this is the case.

**(c)** About how many times did the number of cases in the US double in that 10-day period? How about Egypt in that 21-day period?

[7] Data from Johns Hopkins University, downloaded from https://github.com/CSSEGISandData/COVID-19 accessed on June 25, 2020.

[8] Data from Johns Hopkins University, downloaded from https://github.com/CSSEGISandData/COVID-19 accessed on April 26, 2020.