Introduction

P and N functions. The cumulative total number of Covid-19 cases confirmed up to and including day t is denoted by $P(t)$. We allow $t = 0$ to correspond to any given date.

The number of new cases confirmed on N on day t is given by

$$N(t) = P(t) - P(t-1).$$

We sometimes also call $N(t)$ the...

- daily increase in cases on day t.
- daily cases on day t.
- new cases.

Note that with these definitions

$$\begin{aligned} P(t) &= P(t-1) + N(t) \\ &= (\text{Total cases confirmed up to and including day } t) + (\text{New cases confirmed on day } t) \end{aligned}$$

Data sources. All data is sourced from the *COVID-19 Data Repository by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University* accessible at the following link

- https://github.com/CSSEGISandData/COVID-19.

We reference this source as JHU in short.

Getting the data for your own explorations, or for students to explore. Raw data for countries and US states, together with their 7-day smoothing (see below) can be downloaded using a couple of websites that Isaac Flath built for the Consortium.

- https://isaac-flath.shinyapps.io/coronavirus2/
- https://isaac-flath.shinyapps.io/Covid-19/

These two sites also generate plots for any given time range for all the quantities we discuss in the problems we have written (P, N, relative change of P, log plots of P, etc).

Data smoothing. Covid-19 data (specially N-data) is very noisy, so in all the problems we have written we use the 7-day moving average of the data which we compute as follows ($\widetilde{P}(t)$ is the raw time series obtained JHU):

$$\begin{aligned} P(t) &= (\widetilde{P}(t) + \widetilde{P}(t-1) + \ldots + \widetilde{P}(t-6))/7 \\ N(t) &= P(t+1) - P(t) \end{aligned}$$

so, the new cases time series $N(t)$ gets smoothed through the smoothing of $P(t)$ and does not get smoothed separately. This makes the relation $N(t) = P(t+1) - P(t)$ continue to hold after the smoothing. We also make sure that we pick up JHU data well before $t = -7$, so the data is smoothed using the above formula even for $t = 0$ using the previous 7 data points in the JHU time series.

Figures 1 and 2 show the effect of this smoothing for the cumulative cases P and new cases N for Romania, with t in days since March 1, 2020 (thick blue is the smoothed data, thin black is the raw data). A side product of this smoothing is that data gets shifted forward (to the left) by 3.5 days.

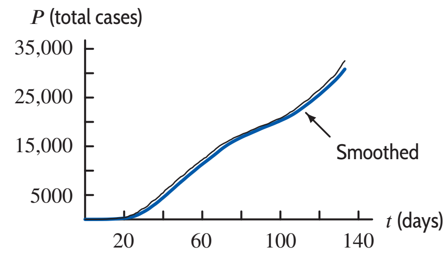

Figure 1

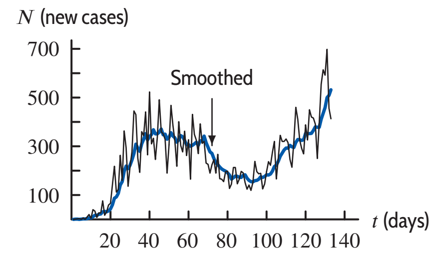

Figure 2

We will not use raw data $\widetilde{P}(t)$ (or $\widetilde{N}(t) = \widetilde{P}(t+1) - \widetilde{P}(t)$) in what follows unless explicitly stated otherwise.

Expected shape of P and N. The Covid-19 outbreaks have occurred in waves in many countries. Each wave has an N-plot which is bell-shaped and that will correspond to a sigmoid shaped P-curve.

Subsequent waves stack on top of one another and result in N-plots that look like a sum of bell-shaped curves (which may or may not drop all the way down to zero between the bells), and this results in a P-plot that will look like a sequence of stacked sigmoid s-curves. Figures 3 and 4 are the P and N plots for Australia between March 1 and August 16, 2020.

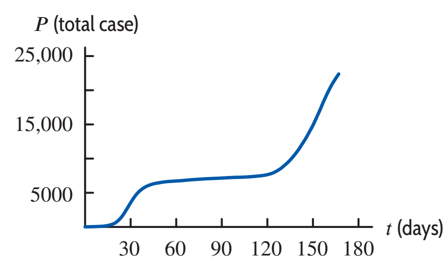

Figure 3

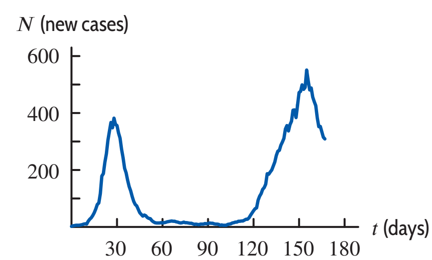

Figure 4

Of course, the outbreaks in many countries do not look so distinctly as a sum of waves. Besides possible data collecting issues or changes in data collecting strategies, drastic measures implemented to control the outbreak have noticeable effects on the way the outbreak developed. Figures 1 and 2 are good examples of expected shapes P and N plots when the waves are not separate.

N is the derivative of P, P is the integral of N Even though one informally thinks that $N = dP/dt$, and so also $\int_a^b N(t) = P(b) - P(a)$, the integral relation must be interpreted with care. Specifically, since $N(t) = P(t+1) - P(t)$ is the forward discrete difference, then the integral relation between P and N is given by the discrete version of the FTC (note the forward shift in the right endpoint):

$$N(a) + N(a+1) \ldots + N(b) = P(b+1) - P(a)$$

One way to gain clarity is to interpret the integral $\int_a^b N\,dt$ as a left hand sum with $\Delta t = 1$. This gives

$$\int_a^b N(t)dt \approx \underbrace{N(a) + N(a+1) + \ldots + N(b-1)}_{\text{left hand sum}} = P(b) - P(a)$$

which does make the relation $\int_a^b N(t)dt \approx P(b) - P(a)$ hold.

Sample problems - Integral Calculus - see the Instructor notes package for more

1. Table 1 shows the confirmed new cases of Covid-19 in New Zealand corresponding to the time period from March 23, 2020 to April 5, 2020 when the country reached its peak number of new confirmed cases. New Zealand had approximately 290 beds available in inten-

sive care units across the country.[1] To simplify the situation we assume that the distribution of new cases and the ICU beds are distributed proportional to the population density.

Table 1

Date	Confirmed New Cases
3/23	20
3/24	27
3/25	36
3/26	47
3/27	57
3/28	59
3/29	69
3/30	71
3/31	72
4/1	73
4/2	71
4/3	72
4/4	75
4/5	74

(a) Let $N(t)$ denote the confirmed daily increase in cases on day t and $P(t)$ denote the total number of confirmed cases on day t of the covid outbreak in New Zealand and assume that the first date of the outbreak in New Zealand, $t = 1$, was March 23, 2020. Write a definite integral that corresponds to the confirmed total cases that occurred between March 23 and April 5, 2020. Approximate the value of the integral.

(b) If every infected person would need intensive care as soon as their infection is detected, and the average stay in the ICU was 4 days would the country be in danger of running out of ICU beds in this time period?

(c) How would your answer change if only 80% of the confirmed cases needed ICU care in this time period?

[1] https://www.anzics.com.au/wp-content/uploads/2020/02/2018-CCR-Report.pdf accessed on May 23, 2020.