**Linear and Exponential Growth: Carbon Dioxide**

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More climate change resources at: https://mcwg.github.io/climate/

Note: Solutions with data from 1950 from [www.earth-policy.org/datacenter/xls/book\_tgt\_climate\_12.xlsx](http://www.earth-policy.org/datacenter/xls/book_tgt_climate_12.xlsx)

1. Download the annual mean CO2 from Mauna Loa between 1959 and 2020 at <http://www.esrl.noaa.gov/gmd/ccgg/trends/index.html> and make a scatter plot with years since 1950 along horizontal axis.

ANS: Yours will be slightly different as I have added data from 1950-1958. But your answers will be very close! Using data from 1950 onward we have

1. Fit a **linear function** to the scatter plot and show the equation.

ANS: Again yours will be slightly different (but not much) as this graph is using data from 1950 onward:

1. Interpret the slope in terms of CO2 and time.

ANS: Using data from 1950 onward: The 1.47 is the rate at which CO2 is increasing in parts per million (ppm) per year.

1. Interpret the intercept in terms of CO2 and time.

ANS: Using data from 1950 onward: 299.34 ppm is the predicted value of the CO2 when $t=0$, that is in 1950.

1. Fit an **exponential function** to the scatter plot and show the equation.

ANS: Using data from 1950 onward:

1. Interpret the coefficient in the exponent in terms of CO2 and time.

ANS: Using data from 1950 onward: The coefficient in the exponent, 0.0045 = 0.45% tells us that CO2 is growing at a continuous rate per year of 0.45% of whatever is present at the time. It is the relative yearly growth rate.

1. Interpret the constant in front of the exponential in terms of CO2 and time.

ANS: Using data from 1950 onward: The 297.82 ppm is the predicted value of the CO2 when $t=0$, that is in 1950.

1. Is the slope of the linear function higher or lower than the rate of increase of the CO2 at the start of the time-period? At the end of the time-period?

ANS: Start: Slope of linear function, 1.47, is higher than rate of increase of CO2 at start of period.

End: Slope 1.47 is lower than rate of increase of CO2 at end of the period.

How can we see this? Look at shape of curves and data: Shape of data is “bending upward” as growth rate has been increasing. Because it is attempting to ft the data, the exponential is also “bending upward”. Thus it has a smaller slope than line at the start and larger at the end.

1. Is the relative growth rate of the linear function higher or lower than the rate of increase of the CO2 at the start of the time-period? At the end of the time-period?

ANS: Start: Relative growth rate, 0.45%, is higher

End: Relative growth rate, 0.45%, is lower

1. Which model fits the data best? (Verbal answer.)

ANS: Both are good, but neither perfect: CO2 is increasing faster than *both* linear and exponential.