Note: Some problems in this document will be available in *Calculus, 9th edition, Hughes Hallett et al., John Wiley & Sons*, to be published in 2027.

Problems to be solved with AI

1. The Dow Jones Industrial Average (DJIA) is one measurement of how the US economy is performing, so big rates of change in the DJIA and the reasons behind them are of interest to some economists and people in the financial industry.
   - **(a)** Use an LLM to sketch a graph of the daily value of the DJIA since 1st January 2018 through 1st January 2026. You can access the data here.[1]
   - **(b)** The DJIA fluctuates over time, but there was a particularly steep decline leading to its minimum value over this timeframe. Use the graph to approximate the dates when this decline started and ended, and then ask the LLM to give precise dates and the value of the DJIA on those dates.
   - **(c)** Find the rate of change of the DJIA over the interval you found in part (b) and write a sentence interpreting the value.
   - **(d)** A decline of 20% or more in the stock market is called a bear market and is often triggered by major external events. What event caused this bear market?
   - **(e)** Use an LLM to identify any other bear markets (at least 20% drop) on the graph. For each bear market, find the value of the DJIA at the beginning and end of the decline, the dates on which these values occurred, and the number of days between the beginning and ending dates.
   - **(f)** Find the rate of change of the DJIA from the beginning to the end of each bear market and write a sentence interpreting the value.

2. Cross-sectional images, such as CT or MRI, can be used to approximate total lung capacity of a patient.
   - **(a)** Figure 1 shows a CT scan of a cross-section of someone's lungs, the dark internal areas surrounded by white. Estimate the area inside the lungs using the fact that the scan region, which is the gray rectangle bounded by black vertical bands in the image, is 37.4 cm wide. The vertical scale is the same as the horizontal.
   - **(b)** Use an LLM to approximate the area inside the lungs by giving it a large version of the image (download here[2]). Make sure to tell the LLM that the scanned region is 37.4 cm wide and that the vertical and horizontal scales are the same. Compare the result to your estimate in part (a) and ask the LLM to show you an image of the areas it found to check its work.
   - **(c)** What is the approximate volume of the lung capacity for a 3-cm slice if all its cross sections are equal to Figure 1?
   - **(d)** Explain how to approximate the total lung capacity with 11 CT scans starting 3 cm from the bottom of the lung and taken at 3 cm intervals.
   - **(e)** Use an LLM to analyze the 11 CT scans, found here,[3] to give the lung capacity for each slice, and then approximate the volume assuming each slice is 3 cm high.

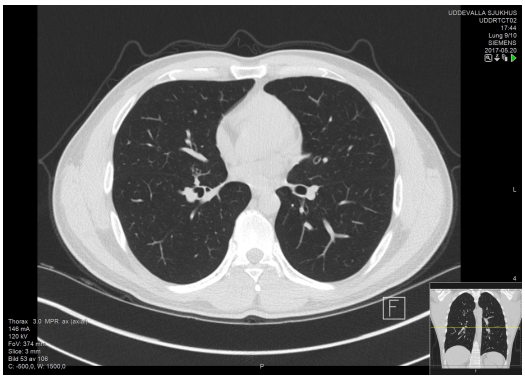


Figure 1

[1] https://fred.stlouisfed.org/series/DJIA

[2] `https://datatrove-edu.github.io/entries/total-lung-capacity/53.jpg`, originally from `https://commons.wikimedia.org/wiki/Scrollable_high-resolution_computed_tomography_images_of_a_normal_thorax`, accessed in January 2026.

[3] `https://datatrove-edu.github.io/entries/total-lung-capacity/11LungCTSlices.zip`, originally from `https://commons.wikimedia.org/wiki/Scrollable_high-resolution_computed_tomography_images_of_a_normal_thorax`, accessed in January 2026.

3. Due to remarkable driving, Lewis Hamilton claimed first place at the Silverstone Grand Prix in 2020 despite getting a puncture during the final lap. Figure 2 shows Hamilton's speed V_1 and V_2, in kilometers per hour, as a function of distance traveled m in kilometers from the start of the track over the final two laps respectively.

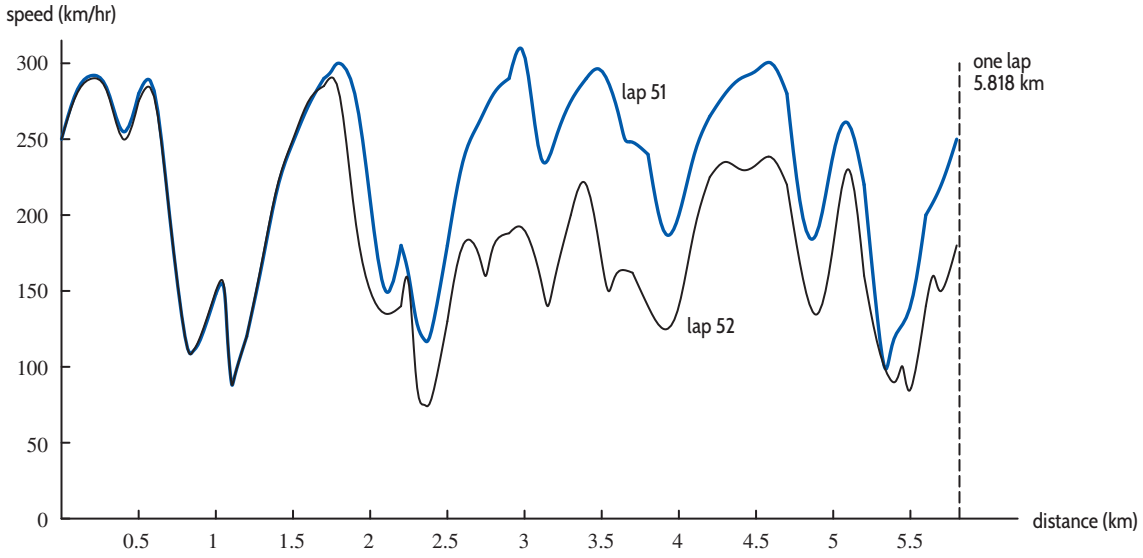


Figure 2

(a) Ask an LLM to use the graph to construct a Riemann sum to estimate $\int_0^{5.8} V_1\, dm$, the integral of velocity V_1 over the second-to-last lap. Use a right-hand sum with subdivisions of length 0.1 km and make sure the LLM uses the graph to estimate the correct value of velocity at the right-hand point of every subinterval. Ask the LLM to give you a plot with the shaded rectangles it used to estimate the integral to make sure that it correctly extracted the data from the graph and that it used the correct rectangles.
(b) Use (a) to estimate Hamilton's average velocity over the second-to-last lap.
(c) Repeat (a) and (b) to estimate Hamilton's average velocity on the last lap.
(d) Use your work to estimate how much longer Hamilton took to complete the final lap than the previous lap.

4. The number of deaths each week in the US is sinusoidal and roughly follow a sine curve.

(a) Using weekly US CDC death data from 2017–2019 found here,[4] use an LLM to fit a sinusoidal model representing a typical year, where t is the number of weeks since the first week of January ($0 \le t < 52$). Plot the model together with the data for each of the three years to check that your model looks reasonable. If not, push back with the LLM.
(b) Which week does you model predict is the deadliest week of the year?
(c) Use your model to calculate the number of expected deaths during the week of April 11th.
(d) Excess deaths are the difference between observed deaths and expected deaths. The CDC reports 79,101 deaths the week of April 11th, 2020. Find the number of excess deaths over this week and ask the LLM to add the April 11th, 2020 point to the graph to illustrate the excess deaths.

[4] `https://datatrove-edu.github.io/entries/deaths-per-week-us/us_weekly_all_cause_deaths_2017-2019.csv`, processed from raw data from the National Center for Health Statistics at `https://data.cdc.gov/d/xkkf-xrst`, accessed in June 2026.